

Aether compactification

Sean M. Carroll¹ and Heywood Tam¹¹*California Institute of Technology, Pasadena, California 91125, USA*

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We propose a new way to hide large extra dimensions without invoking branes, based on Lorentz-violating tensor fields with expectation values along the extra directions. We investigate the case of a single vector aether field on a compact circle. In such a background, interactions of other fields with the aether can lead to modified dispersion relations, increasing the mass of the Kaluza-Klein excitations. The mass scale characterizing each Kaluza-Klein tower can be chosen independently for each species of scalar, fermion, or gauge boson. No small-scale deviations from the inverse square law for gravity are predicted, although light graviton modes may exist.

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I. INTRODUCTION

If spacetime has extra dimensions in addition to the four we perceive, they are somehow hidden from us. For a long time, the only known way to achieve this goal was the classic Kaluza-Klein scenario: compactify the dimensions on a manifold of characteristic size $\sim R$. Momentum in the extra dimensions is then quantized in units of R^{-1} , giving rise to a Kaluza-Klein tower of states; if R is sufficiently small, the extra dimensions only become evident at very high energies. More recently, it has become popular to consider scenarios in which standard model fields are localized on a brane embedded in a larger bulk [1–4]. In this picture, the extra dimensions are difficult to perceive because we cannot get there.

In this paper we consider a new way to keep extra dimensions hidden, or more generally to affect the propagation of fields along directions orthogonal to our macroscopic dimensions: adding Lorentz-violating tensor fields (aether) with expectation values aligned along the extra directions. Interactions with the aether modify the dispersion relations of other fields, leading (with appropriate choice of parameters) to larger energies associated with extra-dimensional momentum.¹ We should emphasize that we have no underlying reason for choosing any particular values of the relevant parameters; in particular, obtaining very large mass splittings requires unnaturally large parameters. Mass splittings that are different for different species are, however, generic predictions of the model.

This scenario has several novel features. Most importantly, it allows for completely different spacings in the

Kaluza-Klein towers of each species. If the couplings are chosen universally, the extra mass given to fermions will be twice that given to bosons. There will also be new degrees of freedom associated with fluctuations of the aether field itself; these are massless Goldstone bosons from the spontaneous breaking of Lorentz invariance, but can be very weakly coupled to ordinary matter. There is a sense in which the effect of the aether field is to distort the background metric, but in a way that is felt differently by different kinds of fields. The extra dimensions can be “large” if the expectation value of the aether field is much larger than the inverse coupling. In contrast to brane-world models, we expect no deviation from Newton’s inverse square law even if the extra dimensions are as large as a millimeter, as the gravitational source will be distributed uniformly throughout the extra dimensions rather than confined to a brane. The model has no obvious connection to the hierarchy problem; indeed, hiding large dimensions requires the introduction of a new hierarchy. New physical phenomena associated with the scenario deserve more extensive investigation.

II. AETHER

For definiteness, consider a five-dimensional flat spacetime with coordinates $x^a = \{x^\mu, x^5\}$ and metric signature $(- + + + +)$. The fifth dimension is compactified on a circle of radius R . The aether is a spacelike five-vector u^a , and we can define a “field strength” tensor

$$V_{ab} = \nabla_a u_b - \nabla_b u_a. \quad (1)$$

This field is not related to the electromagnetic vector potential A_a or its associated field strength $F_{ab} = \nabla_a A_b - \nabla_b A_a$, nor will the dynamics of u^a respect a U(1) group of gauge transformations. Rather, the aether field will be fixed to have a constant norm, with an action

$$S = M_* \int d^5x \sqrt{-g} \left[-\frac{1}{4} V_{ab} V^{ab} - \lambda(u_a u^a - v^2) + \sum_i \mathcal{L}_i \right]. \quad (2)$$

¹After this paper was completed, we became aware of closely related work by Rizzo [5]. He enumerated a complete set of five-dimensional Lorentz-violating operators that preserve Lorentz invariance in 4D, and calculated their effect on the spectrum of the Kaluza-Klein tower. In contrast, our starting point is the expectation value of a dynamical aether field, and its lowest-order couplings to ordinary matter. The modified dispersion relations we derive recover in large measure Rizzo’s phenomenological results.

The \mathcal{L}_i 's are various interaction terms to be considered below, and M_* is an overall scaling parameter. Note that λ is not a fixed parameter, but a Lagrange multiplier enforcing the constraint

$$u^a u_a = v^2. \quad (3)$$

We choose conventions such that u^a has dimensions of mass. The equation of motion for u^a , neglecting interactions with other fields for the moment, is

$$\nabla_a V^{ab} + v^{-2} u^b u_c \nabla_a V^{cd} = 0, \quad (4)$$

where we have used the equations of motion to solve for λ . Any configuration for which $V_{ab} = 0$ everywhere will solve this equation. In particular, there is a background solution of the form

$$u^a = (0, 0, 0, 0, v), \quad (5)$$

so that the aether field points exclusively along the extra direction. We will consider this solution for most of this paper.

Constraints on four-dimensional Lorentz violation via couplings to Standard Model fields have been extensively studied [6–9]. The dynamics of the (typically timelike) aether fields themselves and their gravitational effects have also been considered [10–19]. More recently, attention has turned to the case of spacelike vector fields, especially in the early universe [20,21].

The particular form of the Lagrangian (2) is chosen to ensure stability of the theory; for spacelike vector fields, a generic set of kinetic terms would generally give rise to negative-energy excitations. This specific choice propagates two positive-energy modes: one massless scalar, and one massless pseudoscalar [21]. For purposes of this paper we will not investigate the fluctuations of u^a in any detail. Although the modes are light, their couplings to standard model fields can be suppressed. Nevertheless, we expect that traditional methods of constraining light scalars (such as limits from stellar cooling) will provide interesting bounds on the parameter space of these models.

III. ENERGY-MOMENTUM AND COMPACTIFICATION

A crucial property of aether fields is the dependence of their energy density on the spacetime geometry. The energy-momentum tensor takes the form

$$T_{ab} = V_{ac} V_b^c - \frac{1}{4} V_{cd} V^{cd} g_{ab} + v^{-2} u_a u_b u_c \nabla_d V^{cd}. \quad (6)$$

In particular, T_{ab} vanishes when V_{ab} vanishes, as for the constant field configuration in flat space (5). The nonvanishing expectation value for the aether field does not by itself produce any energy density. In the context of an extra dimension, this implies that the aether field will not provide a contribution to the effective potential for the radion, so the task of stabilizing the extra dimension must be left to other mechanisms.

When the background spacetime is not Minkowski, however, even a “fixed” aether field can give a nonvanishing energy-momentum tensor. In [10] it was shown that a timelike aether field would produce an energy density proportional to the square of the Hubble constant, while in [20] a spacelike aether field was shown to produce an anisotropic stress. We should therefore check that an otherwise quiescent aether field oriented along an extra dimension does not create energy density when the four-dimensional geometry is curved.

Consider a factorizable geometry with an arbitrary four-dimensional metric and a radion field $b(x^\sigma)$ parametrizing the size of the single extra dimension,

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + b(x)^2 dx_5^2, \quad (7)$$

where x here stands for the four-dimensional coordinates x^σ . In any such spacetime, there is a background solution

$$u^a = \left(0, 0, 0, 0, \frac{v}{b(x)}\right). \quad (8)$$

It is straightforward to verify that this configuration satisfies the equation of motion (4), as well as the constraint (3), even though V_{ab} does not vanish:

$$V_{\mu 5} = -V_{5\mu} = v \nabla_\mu b. \quad (9)$$

We can then calculate the energy-momentum tensor associated with the aether:

$$\begin{aligned} T_{\mu\nu}^{(u)} &= \frac{v^2}{b^2} \left(\nabla_\mu b \nabla_\nu b - \frac{1}{2} g_{\mu\nu} \nabla_\sigma b \nabla^\sigma b \right), \\ T_{\mu 5}^{(u)} &= 0, \\ T_{55}^{(u)} &= v^2 (\nabla_\sigma \nabla^\sigma b - \frac{1}{2} \nabla_\sigma b \nabla^\sigma b). \end{aligned} \quad (10)$$

The important feature is that $T_{ab}^{(u)}$ will vanish when $\nabla_\mu b = 0$. As long as the extra dimension is stabilized and the aether takes on the configuration (8), there will be no contributions to the energy-momentum tensor; in particular, neither the expansion of the universe nor the spacetime geometry around a localized gravitating source will be affected.

IV. SCALARS

We now return to flat spacetime ($g_{ab} = \eta_{ab}$) and consider the effects of interactions of the aether on various types of matter fields, beginning with a real scalar ϕ . We impose a \mathbb{Z}_2 symmetry, $u^a \rightarrow -u^a$. The Lagrangian with the lowest-order coupling is then

$$\mathcal{L}_\phi = -\frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{2\mu_\phi^2} u^a u^b \partial_a \phi \partial_b \phi, \quad (11)$$

with a corresponding equation of motion

$$\partial_a \partial^a \phi - m^2 \phi = \mu_\phi^{-2} \partial_a (u^a u^b \partial_b \phi). \quad (12)$$

Expanding the scalar in Fourier modes,

$$\phi \propto e^{ik_a x^a} = e^{ik_\mu k^\mu + ik_5 x^5}, \quad (13)$$

yields a dispersion relation

$$-k_\mu k^\mu = m^2 + (1 + \alpha_\phi^2)k_5^2, \quad (14)$$

where

$$\alpha_\phi = v/\mu_\phi. \quad (15)$$

Note that with our metric signature, $-k_\mu k^\mu = \omega^2 - \vec{k}^2$.

This simple calculation illustrates the effect of the coupling to the spacelike vector field. Compactifying the fifth dimension on a circle of radius R quantizes the momentum in that direction, $k_5 = n/R$. In standard Kaluza-Klein theory, this gives rise to a tower of states of masses $m_{\text{KK}}^2 = m^2 + (n/R)^2$. With the addition of the aether field, the mass spacing between different states in the KK tower is enhanced,

$$m_{\text{AC}}^2 = m^2 + (1 + \alpha_\phi^2)\left(\frac{n}{R}\right)^2. \quad (16)$$

The parameter α_ϕ is a ratio of the aether vev to the mass scale μ_ϕ characterizing the coupling, and could be much larger than unity. If the vev is $v \sim M_P$, and the coupling parameter is $\mu_\phi \sim \text{TeV}$, the masses of the excited modes are enhanced by a factor of 10^{15} . The extra dimension could be as large as $R \sim 1 \text{ mm}$, and the $n = 1$ state would have a mass of order TeV. Admittedly, we have no compelling reason why there should be such a hierarchy between v and μ_ϕ at this point, other than that it is interesting to contemplate.

We will examine the effects of aether compactification on gravitons below, but it is already possible to see that we should not expect any small-scale deviations from Newton's law, even if the extra dimensions are millimeter-sized. Unlike braneworld compactifications, here the sources are not confined to a thin brane embedded in a large bulk; rather, light fields are zero modes, spread uniformly throughout the extra dimensions. Therefore, the gravitational lines of force do not spread out from the source into the higher-dimensional bulk; the sources are still of *codimension* three in space, and gravity will appear three dimensional. There is correspondingly less motivation for considering macroscopic-sized extra dimensions in this scenario, as they would remain undetectable by tabletop experiments.

One may reasonably ask whether it is appropriate to think of such a scenario as a large extra dimension at all, or whether we have simply rescaled the metric in an unusual way. In the Lagrangian (11) alone, the effect of the aether field is simply to modify the metric by a disformal transformation, $g^{ab} \rightarrow g^{ab} + u^a u^b$. There is a crucial difference, however, in that the interaction with the aether vector is generically not universal. Different fields

will tend to have different mass splittings in their Kaluza-Klein towers. Indeed, we shall see that while the splittings for gauge fields follow the pattern of that for scalars, the splittings for fermions are of order α^4 rather than α^2 , and the splittings for gravitons do not involve a mass scale μ at all. Thus, aether compactification is conceptually different from an ordinary extra dimension.

Finally, we point out that if we have not imposed the \mathbb{Z}_2 symmetry, the lowest-order coupling becomes $\mu^{-1} u^a \partial_a \phi$. By integration by parts, this is equivalent to $-\mu^{-1} (\partial_a u^a) \phi$, which vanishes given our background solution for u^a in (5).

V. GAUGE FIELDS

Consider an Abelian gauge field A_a , with field strength tensor F_{ab} . The Lagrangian with the lowest-order coupling to u^a is

$$\mathcal{L}_A = -\frac{1}{4} F_{ab} F^{ab} - \frac{1}{2\mu_A^2} u^a u^b g^{cd} F_{ac} F_{bd}, \quad (17)$$

with equation of motion

$$\partial_a F^{ab} = \mu_A^{-2} (u_c u^b \partial_a F^{ca} - u_c u^a \partial_a F^{cb}). \quad (18)$$

We can decompose this into $b = 5$ and $b = \nu$ components in the background (5):

$$\partial_\mu F^{\mu 5} = 0, \quad (19)$$

$$\partial_\mu F^{\mu \nu} = -(1 + \alpha_A^2) \partial_5 F^{5\nu}, \quad (20)$$

where

$$\alpha_A = v/\mu_A. \quad (21)$$

We can take advantage of gauge transformations $A_a \rightarrow A_a + \partial_a \lambda$ to set $A_5 = 0$. This leaves some residual gauge freedom; we can still transform $A_\mu \rightarrow A_\mu + \partial_\mu \tilde{\lambda}$, as long as $\partial_5 \tilde{\lambda} = 0$. In other words, the zero mode retains all of its conventional four-dimensional gauge invariance.

Choose $A_5 = 0$ gauge, and go to Fourier space, $A^\nu \propto \epsilon^\nu e^{ik_\mu x^\mu + ik_5 x^5}$, where ϵ^ν is the polarization vector. Then (19) and (20) imply

$$k_5 k_\mu \epsilon^\mu = 0, \quad (22)$$

$$[k_\mu k^\mu + (1 + \alpha_A^2)k_5^2] \epsilon^\nu - k^\nu k_\mu \epsilon^\mu = 0. \quad (23)$$

When $k_5 = 0$, we obtain the ordinary dispersion relation for a photon. When k_5 is not zero, (22) implies $k_\mu \epsilon^\mu = 0$, and the dispersion relation is

$$-k_\mu k^\mu = (1 + \alpha_A^2)k_5^2. \quad (24)$$

Precisely as in the scalar case, the Kaluza-Klein masses are enhanced by a factor $(1 + \alpha_A^2)$, although there is no necessary relationship between α_A and α_ϕ . The same reasoning

would apply to non-Abelian gauge fields, through a coupling $u^a u^b \text{Tr}(G_{ac} G_b^c)$.

VI. FERMIONS

Next we turn to fermions, taken to be Dirac for simplicity. Given the symmetry $u^a \rightarrow -u^a$, we might consider a coupling of the form $u^a u^b \bar{\psi} \gamma_a \gamma_b \psi$. But because $u^a u^b$ is symmetric in its two indices, this is equivalent to $u^a u^b \bar{\psi} \gamma_{(a} \gamma_{b)} \psi = u^a u^b \bar{\psi} g_{ab} \psi = v^2 \bar{\psi} \psi$, so this interaction does not violate Lorentz invariance.

The first nontrivial coupling involves one derivative,

$$\mathcal{L}_\psi = i \bar{\psi} \gamma^a \partial_a \psi - m \bar{\psi} \psi - \frac{i}{\mu_\psi^2} u^a u^b \bar{\psi} \gamma_a \partial_b \psi, \quad (25)$$

leading to an equation of motion

$$i \gamma^a \partial_a \psi - m \psi - \frac{i}{\mu_\psi^2} u^a u^b \gamma_a \partial_b \psi = 0. \quad (26)$$

Going to Fourier space as before, we ultimately find a dispersion relation

$$-k^a k_a - \frac{2}{\mu_\psi^2} (u^a k_a)^2 - \frac{1}{\mu_\psi^4} u^a u_a (u^b k_b)^2 = m^2. \quad (27)$$

Plugging in the background (5) and defining

$$\alpha_\psi = v/\mu_\psi, \quad (28)$$

we end up with

$$-k^\mu k_\mu = m^2 + (1 + \alpha_\psi^2)^2 k_5^2. \quad (29)$$

Although the form of this equation is identical to the scalar and gauge-field cases, it is quantitatively different: for large α the enhancement goes as α^4 rather than α^2 . If (in the context of some as-yet-unknown underlying theory) all of the mass scales μ are similar, we would expect a much larger mass splitting for fermions in an aether background than for bosons.

Similar to the scalar case, if we do not impose the \mathbb{Z}_2 symmetry, we are led to consider the following two lower order couplings: $u_a \bar{\psi} \gamma^a \psi$ and $\frac{i}{\mu} u^a \bar{\psi} \partial_a \psi$. Following the same procedure as before, the first term leads to the dispersion relation

$$-k_\mu k^\mu = m^2 + v^2 + k_5^2 + 2v k_5 = m^2 + (v + k_5)^2. \quad (30)$$

As usual, coupling to u^a enhances the mass spacing of the KK tower, but now the spacing will depend on the direction of the 5th-dimensional momentum as well as its magnitude.

Meanwhile, the second term leads to the dispersion

$$-k_\mu k^\mu = m^2 - 2m\alpha k_5 + (1 + \alpha^2) k_5^2 \quad (31)$$

$$= (m - \alpha k_5)^2 + k_5^2, \quad (32)$$

where $\alpha = v/\mu$. Interestingly, if $(1 + \alpha^2)/\alpha < 2mR$, this coupling results in a reduction in m^2 for small n . However, it can be checked that these negative mass corrections are never sufficiently large to lead to tachyons. For n large, the mass spacing is enhanced, as usual.

VII. GRAVITY

The aether field can couple nonminimally to gravity through an action

$$S = M_* \int d^5 x \sqrt{-g} \left[\frac{M_P^2}{2} R + \alpha_g u^a u^b R_{ab} \right], \quad (33)$$

where M_P is the 4-dimensional Planck scale and α_g is dimensionless. The gravitational equation of motion takes the form

$$G_{ab} = \frac{\alpha_g}{2M_P^2} W_{ab}, \quad (34)$$

where $G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}$ and

$$W_{ab} = R_{cd} u^c u^d g_{ab} + \nabla_c \nabla_a (u_b u^c) + \nabla_c \nabla_b (u_a u^c) - \nabla_c \nabla_d (u^c u^d) g_{ab} - \nabla_c \nabla^c (u_a u_b). \quad (35)$$

Now we consider small fluctuations of the metric,

$$g_{ab} = \eta_{ab} + h_{ab}. \quad (36)$$

The choice of background field $u^a = (0, 0, 0, 0, v)$ spontaneously breaks diffeomorphism invariance, so not all coordinate transformations are open to us if we want to preserve that form. Under an infinitesimal coordinate transformation parameterized by a vector field, $x^a \rightarrow \bar{x}^a = x^a + \xi^a$, the metric fluctuation and aether change by $h_{ab} \rightarrow h_{ab} + \partial_a \xi_b + \partial_b \xi_a$ and $u^a \rightarrow u^a + \partial_5 \xi^a$. Therefore, we should limit our attention to gauge transformations satisfying $\partial_5 \xi^a = 0$. We can, for example, set $h_{\mu 5} = 0$. We then still have residual gauge freedom in the form of ξ^μ , as long as $\partial_5 \xi^\mu = 0$. This amounts to the usual 4-d gauge freedom for the massless four-dimensional graviton.

Taking advantage of this gauge freedom, we can partly decompose the metric perturbation as

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \Psi \eta_{\mu\nu}, \quad h_{55} = \Phi, \quad (37)$$

where $\eta^{\mu\nu} \bar{h}_{\mu\nu} = 0$. In this decomposition, $\bar{h}_{\mu\nu}$ represents propagating gravitational waves, Ψ represents Newtonian gravitational fields, and Φ is the radion field representing the breathing mode of the extra dimension. The zero mode of this field is a massless scalar coupled to matter with gravitational strength; in a phenomenologically viable model, it would have to be stabilized, presumably by bulk matter fields. The Einstein tensor becomes

$$G_{\mu\nu} = \frac{1}{2}[-\partial_\lambda \partial^\lambda \bar{h}_{\mu\nu} - \partial_5^2 \bar{h}_{\mu\nu} + \partial_\mu \partial^\lambda \bar{h}_{\lambda\nu} + \partial_\nu \partial^\lambda \bar{h}_{\lambda\mu} - 2\partial_\mu \partial_\nu \Psi - \partial_\mu \partial_\nu \Phi - (\partial^\rho \partial^\sigma \bar{h}_{\rho\sigma} - 2\partial_\lambda \partial^\lambda \Psi + 3\partial_5^2 \Psi - \partial_\lambda \partial^\lambda \Phi)\eta_{\mu\nu}], \quad (38)$$

$$G_{\mu 5} = \frac{1}{2}(\partial_5 \partial^\lambda \bar{h}_{\lambda\mu} - 3\partial_\mu \partial_5 \Psi), \quad (39)$$

$$G_{55} = \frac{1}{2}(-\partial^\rho \partial^\sigma \bar{h}_{\rho\sigma} + 3\partial_\lambda \partial^\lambda \Psi), \quad (40)$$

and (35) is

$$W_{\mu\nu} = v^2(\partial_5^2 \bar{h}_{\mu\nu} - 3\partial_5^2 \Psi \eta_{\mu\nu} - \partial_5^2 \Phi \eta_{\mu\nu}), \quad (41)$$

$$W_{\mu 5} = v^2 \partial_\mu \partial_5 \Phi, \quad (42)$$

$$W_{55} = -2v^2(2\partial_5^2 \Psi + \partial_5^2 \Phi). \quad (43)$$

We have already argued that there will be no macroscopic deviations from Newton's law on the scale of the extra-dimensional radius R , because the zero-mode fields are distributed uniformly through the extra dimensions. However, we can also inquire about the Kaluza-Klein tower of propagating gravitons. To that end, we set $\Phi = 0 = \Psi$ and consider transverse waves, $\partial^\lambda \bar{h}_{\lambda\mu} = 0$. The gravity equation (34) becomes

$$-\frac{1}{2}\partial_c \partial^c \bar{h}_{\mu\nu} = \frac{\alpha_g v^2}{2M_P^2} \partial_5^2 \bar{h}_{\mu\nu}. \quad (44)$$

This implies a dispersion relation

$$-k_\mu k^\mu = \left(1 + \frac{\alpha_g v^2}{M_P^2}\right) k_5^2. \quad (45)$$

As before, there is an altered dispersion relation for modes with bulk momentum. However, the dimensionless coupling α_g appears directly in the Lagrangian, rather than arising as a ratio $\alpha = v/\mu$. It is therefore consistent to imagine scenarios with $\alpha_g \sim 1$, while the other α_i 's are substantially larger. In that case, KK gravitons will have masses that are close to the conventional expectation, $m \approx n/R$, even while other fields are much heavier. In the scenario with a single extra dimension, the underlying quantum-gravity scale $M_{\text{QG}}^3 = M_* M_P^2$ will still be substantially larger than a TeV, and we do not expect graviton production at colliders; but such a phenomenon might be important in extensions with more than one extra dimension.

VIII. CONCLUSIONS

The presence of Lorentz-violating aether fields in extra dimensions introduces novel effects into Kaluza-Klein compactification schemes. Interactions with the aether alter the relationship between the size of the extra dimensions and the mass splittings within the KK towers. With appropriately chosen parameters, modes with extra-dimensional momentum can appear very heavy from a four-dimensional perspective, even with relatively large extra dimensions.

A number of empirical tests of this idea suggest themselves. The most obvious is the possibility of KK towers with substantially different masses for different species. While scalar and gauge-boson mass splittings follow a similar pattern, fermions experience greater enhancement, while gravitons can naturally be less massive. In addition, although we have not considered the prospect carefully in this paper, oscillations of the aether field itself are potentially detectable. Their couplings will be suppressed by the mass scales μ_i , without being enhanced by the vev v ; nevertheless, searches for massless Goldstone bosons should provide interesting constraints on the parameter space.

Our investigation has been phenomenological in nature; we do not have an underlying theory of the aether field, nor any natural expectation for the magnitudes of the parameters v , μ_i , and α_g . The possibility of a hidden millimeter-sized dimension requires a substantial hierarchy, $v/\mu_i \sim 10^{15}$; even in the absence of such large numbers, however, interactions with the aether may lead to subtle yet important effects. It would certainly be interesting to have a deeper understanding of the possible origin of these fields and couplings.

Numerous questions remain to be addressed. We considered a vector field in a single extra dimension, but higher-rank tensors in multiple dimensions should lead to analogous effects. It would also be interesting to study the gravitational effects of the aether fields themselves in non-trivial spacetime backgrounds. The idea of modified extra-dimensional dispersion relations in the presence of Lorentz-violating tensor fields opens up a variety of possibilities that merit further exploration.

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- [1] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998).
- [2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998).
- [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
- [4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999).
- [5] T. G. Rizzo, J. High Energy Phys. 09 (2005) 036.
- [6] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
- [7] S. R. Coleman and S. L. Glashow, Phys. Lett. B **405**, 249 (1997).
- [8] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998).
- [9] J. Soda and S. Kanno, arXiv:gr-qc/0612069.
- [10] S. M. Carroll and E. A. Lim, Phys. Rev. D **70**, 123525 (2004).
- [11] V. A. Kostelecky and S. Samuel, Phys. Rev. D **40**, 1886 (1989).
- [12] T. Jacobson and D. Mattingly, Phys. Rev. D **64**, 024028 (2001).
- [13] C. Eling and T. Jacobson, Phys. Rev. D **69**, 064005 (2004).
- [14] V. A. Kostelecky, Phys. Rev. D **69**, 105009 (2004).
- [15] T. Jacobson and D. Mattingly, Phys. Rev. D **70**, 024003 (2004).
- [16] E. A. Lim, Phys. Rev. D **71**, 063504 (2005).
- [17] R. Bluhm, S. H. Fung, and V. A. Kostelecky, Phys. Rev. D **77**, 065020 (2008).
- [18] M. V. Libanov and V. A. Rubakov, J. High Energy Phys. 08 (2005) 001.
- [19] H. C. Cheng, M. A. Luty, S. Mukohyama, and J. Thaler, J. High Energy Phys. 05 (2006) 076.
- [20] L. Ackerman, S. M. Carroll, and M. B. Wise, Phys. Rev. D **75**, 083502 (2007).
- [21] T. R. Dulaney, M. I. Gresham, and M. B. Wise, Phys. Rev. D **77**, 083510 (2008).